

# THROUGHPUT OF A WIRELESS LAN ACCESS POINT IN PRESENCE OF NATURAL HIDDEN TERMINALS AND CAPTURE EFFECTS

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## Abstract

In a wireless LAN, even if the coverage of the access point (AP) and the mobile terminals are the same, the hidden terminal problem can not be avoided. If the AP and all the mobile terminals have the same coverage with a radius  $R$  there will always be terminals in the coverage area of the AP which are hidden to one another. This paper introduces this phenomenon, that we refer to as the natural hidden terminal problem, and analyzes the throughput of the AP under these circumstances with various conditions. The analysis includes the effects of the difference in the coverage of the AP and the mobile terminals as well as the capture effect caused by the near-far problem.

## I. Introduction

The analysis of the real throughput of an AP in a wireless LAN is rather complicated because it involves natural hidden terminal and capture effects. Assuming that the coverage area of the AP and the mobile terminals are the same, there is no guarantee that all the terminals in the coverage area of the AP can hear one another. Therefore the throughput of any AP is degraded from its expected value due to the hidden terminals - natural to radio operation in a hub based network. In real installations, the coverage area of the AP is usually larger than the mobile terminals, because the AP is installed in a selected location to optimize the coverage (high on the walls or on the ceiling), that will increase the negative impacts of hidden terminal problem. The main objective of this paper is to formulate a frame work for the analysis of this problem. In this formulation we also include the effects of capture caused by the near-far problem.

Three factors that are effective in throughput degradation in wired environment are propagation delay, user's idle period (not transmitting), and packet collision because of overlapping of transmissions from multiple users. In wireless environment, hidden terminal effect causes additional degradation. In this case, some terminals can not sense the

carrier of the transmitting terminal and their transmitted packets collide with other packets. Limited antenna range and shadowing are two major causes for hidden terminal degradation. Another phenomenon impacting the throughput of a radio network is capture. In radio channels, sometimes collision of two packets may not destroy both packets. Because of signal fading or near-far effect, packets from different transmitting stations can arrive with different power levels, and strongest packet may survive a collision. The capture effect increases the throughput of the radio network.

Recently, there have been efforts to analyze the effects of capture and hidden terminals in packet radio systems using various assumptions. Hidden terminal effects was first analyzed in [3] for different type of CSMA protocol used in rapidly moving packet radio networks for military applications and busy tone signaling was suggested for eliminating the hidden terminal problem. Reference [7] addresses a form of the hidden terminal problem for CSMA with 4-way handshaking as the point coordination function access of IEEE 802.11 wireless LANs. In [4] and [7], using the probability of hiding a terminal from others, the performance of the CSMA is analyzed. Another approach has been taken in [13] that considers imperfect carrier sensing.

The literature in analysis of capture effect is rather rich [1,2]. The analysis in [10] forms the ratio of the power of a test packet to the total sum of all other colliding packets and compares the results with the capture parameter. If the ratio is higher than the capture parameter, the test packet survives the collision. The channel is assumed to be Rayleigh fading, and the analysis is presented for a general spatial distribution function of the terminals. The reference [12] has done performance analysis of ALOHA with markov model and considered the capture model as in [10]. The analysis in [11] calculates the throughput for CSMA protocol in a Rayleigh fading channel with the capture model in [10] and [12]. Reference [9] performs the same analysis with uniform and bell-shaped distribution for the terminals. The above

approaches isolate the collision mechanism from the modulation, coding technique, and the effect of the additive white Gaussian noise by defining a capture parameter based on the ratio of the received powers from various terminals. In reality, the capture of a packet is a random process which is a function of the modulation technique used for transmission, received signal-to-noise ratio, and the length of the packet. Reference [14] has considered the effects of modulation and coding in capture model, and [6] presents an exact calculation for capture considering modulation technique and length of the packet.

This paper focuses on the throughput analysis for CSMA protocol for mobile users distributed in the coverage area of an AP in a wireless LAN environment. The effects of natural hidden terminals and the capture are included in the analysis. Section two introduces the natural hidden terminal phenomenon and analyzes its impact on the throughput of the AP. The mathematical framework introduced in this section is expanded in section three to include the effects of capture. Section four provides the results and discussions and the conclusions is provided in the section five.

## II. Natural Hidden Terminals and the Throughput Analysis

This section introduces the natural hidden terminal problem and develops a mathematical structure to analyze its impact on the throughput of a CSMA network. The AP is located in the center and the mobile terminals are distributed around it according to a given spatial distribution density function. This system has negligible propagation delay, perfect acknowledgments from the AP, and an infinite number of terminals.

The average packet generation rate from all terminals is taken to be  $\lambda$ . The arrival process follows a Poisson distribution and the packet length has been normalized to 1. Therefore, the average number of packets arriving in one packet slot is  $G = \lambda$ . The range of all mobile stations are  $R_{\text{mob}}$  and the range of AP antenna has been considered  $R \geq R_{\text{mob}}$ .

Fig 1. explains the natural hidden terminal problem. Two circles represent the coverage areas of the AP and mobile terminals in the small square area. Mobile terminals located in the shaded area are in the coverage area of the AP but they can not be heard by the mobile terminals in the small square area. The traffic generated from the terminals located in the shaded area is hidden from the terminals in the small square. To calculate the throughput, we divide the main circle (AP coverage area) in  $N$  small areas. The  $i$ -th area has a throughput  $\Delta S_i$  and a traffic  $\Delta G_i$ . Each small division partitions the main cell into 2 areas: the area where the carrier can be sensed and the area where the carrier can not be sensed. If a packet is transmitted successfully from  $i$ -th division, the following two mutually independent conditions should be satisfied:

Condition C1: The packet transmission should be successful (no collision) in area 1.

Condition C2: The packet transmission should be successful (no collision) in area 2.

Therefore we can derive probability of success,  $P_{s_i}$ , for  $i$ -th division as multiplication of probability success under C1 condition and probability success under C2 condition. As shown in [3]

$$P_r(C1) = \frac{e^{-a \cdot G_1}}{G_1 \cdot (1 + 2 \cdot a) + e^{-a \cdot G_1}} \quad (1)$$

$$P_r(C2) = \frac{e^{-G_2 \cdot (1-a)}}{G_2 \cdot (1 + 2 \cdot a) + e^{-a \cdot G_2}} \quad (2)$$

$$P_{s_i} = \frac{\Delta S_i}{\Delta G_i} = P_r(C1) \cdot P_r(C2) \quad (3)$$

When  $N \rightarrow \infty$ ,  $\Delta G_i$  and  $\Delta S_i \rightarrow 0$ , but  $P_{s_i}$  will be close to  $P_s(r)$ , where  $r$  is the distance of  $i$ -th part from the AP. If  $G(r, \theta)$  is the traffic in distance  $r$  and angle  $\theta$  in polar coordination system in our cell, then for a small area  $\Delta A_i$ , the  $G(r, \theta)$  is approximately constant on  $\Delta A_i$  and  $\Delta G_i = G(r, \theta) \cdot \Delta A_i$ . With this definitions, one can calculate the throughput  $S$  based on probability of success as follows:

$$S = \lim_{N \rightarrow \infty} \sum_{i=1}^N \Delta S_i = \lim_{N \rightarrow \infty} \sum_{i=1}^N \Delta G_i \cdot P_{s_i} \quad (4)$$

$$S = \iint_{A_1} G(r, \theta) \cdot P_s(r, \theta) \cdot r \cdot dr \cdot d\theta \quad (5)$$

where  $A_1$  is the AP's coverage area.

If traffic in area 1 is  $G_1$  and traffic in area 2, that is hidden to  $\Delta A_i$ , is  $G_2$ , then  $G_1$  and  $G_2$  will be obtained by the following equations:

$$G_2(r) = \begin{cases} \iint_{A_1} G(r, \theta) \cdot r \cdot dr \cdot d\theta - \iint_{A_2} G(r, \theta) \cdot r \cdot dr \cdot d\theta & r \leq R - R_{\text{mob}} \\ \iint_{A_2} G(r, \theta) \cdot r \cdot dr \cdot d\theta & R - R_{\text{mob}} \leq r \leq R \end{cases} \quad (6)$$

$$G_i = \int_0^{2\pi} \int_0^R G(r, \theta) \cdot r \cdot dr \cdot d\theta \quad (7)$$

$$G_1(r) = G_i - G_2(r) \quad (8)$$

where  $A_2$  is the mobile terminal coverage area, and  $A_{12}$  is coverage area of overlapped region. When the coverage of mobile terminal antenna  $R_{\text{mob}}$  is equal to the coverage of AP antenna, calculation of  $G_2(r)$  will be simplified to

$$G_2(r) = \int_{A_2} G(\alpha, \theta) \cdot \alpha \cdot d\alpha \cdot d\theta$$

With  $G_1(r)$  and  $G_2(r)$ , one can derive  $Ps(r)$ :

$$Ps(r) = \frac{e^{-a(G_1(r))}}{G_1(r) \cdot (1+2 \cdot a) + e^{-a \cdot G_1(r)}} \cdot \frac{e^{-(1-a) \cdot G_2(r)}}{G_2(r) \cdot (1+2 \cdot a) + e^{-a \cdot G_2(r)}} \quad (9)$$

### III. Effects of Capture on the Throughput

Since mobile terminals will generally be at different distances from the AP, their respective received signal powers will not be the same. Therefore, in the event of a collision between the data packets, sometimes it is possible for the receiver to successfully decode the packet with the highest signal strength. Assuming that the received signal powers are more or less constant over a packet duration, the performance improvement in various systems which results from the ability of the receiver to capture a signal has been studied in [6], [9]-[12].

For throughput calculations, probability of success should be obtained based on received signal powers from mobile terminals. A propagation model that defines relationship between the received signal power from the mobile terminal and the distance from base station is required. We assume that the signal power and distance are related by

$$\Gamma(r) = \frac{1}{r^4} \quad (10)$$

where  $r$  is the distance between the mobile terminal and the base station. If signal power of test packet is  $\Gamma_t$  and  $N$  other contending packets have cumulative signal power of  $\sum_{i=1}^N \Gamma_i$ ,

then test packet can be received correctly if

$$\Gamma_t \geq c \cdot \sum_{i=1}^N \Gamma_i \quad (11)$$

where  $c$  is a constant referred to as the capture threshold.

Introducing a capture condition in which successful decoding of a test packet is a probabilistic function of the test signal-to-interference ratio  $Z$  defined by

$$Z_n = \frac{\Gamma_t}{\Gamma_n} \quad 0 \leq Z_n \leq \infty \quad (12)$$

where

$$\Gamma_n = \sum_{j=1}^N \Gamma_j$$

and defining the random variable

$$W = \Gamma_n \quad 0 \leq W < \infty \quad (13)$$

Then, considering that  $\Gamma_t$  and  $\Gamma_n$  are stochastically independent, we can obtain the distribution function of the signal-to-interference ratio for the test packet:

$$F_{Z_n}(z_0, n) = \int_0^{z_0} dz \int_0^{\infty} f_{\Gamma_t}(z \cdot w) \cdot f_{\Gamma_n}(w, n) \cdot w \cdot dw \quad (14)$$

in which  $f_{\Gamma_n}(w, n) = [f_{\Gamma_t}(w)]^{n \otimes}$  is the  $n$  times convolution of the density function of the received signal power.

With a uniform spatial distribution of terminals in the AP coverage area, the distribution of signal-to-interference would be:

$$F_{Z_n}(z_0, n) = \int_0^{z_0} \frac{1}{2} \cdot z^{-\frac{3}{2}} \cdot dz \int_{\frac{1}{z \cdot R^4}}^{\infty} w^{\frac{-1}{2}} \cdot f_{\Gamma_n}(w, n) \cdot dw \quad (15)$$

To analyze under the capture conditions, we are interested in probability of capture with a specific threshold for signal-to-interference  $z_0$ . With  $\frac{\Gamma_t}{\Gamma_n} > z_0$ , the probability

of success is  $1 - F_{Z_n}(z_0, n)$ .

Figure 4 shows the probability of success for uniform spatial distribution of the terminals and  $z_0=1, 2, 4$ , when the number of contending packets vary from 1 to 30.

From mobile station point of view there are two different cases for the busy and idle periods. The first case is when the traffic  $G_1(r) < 1$ , with  $I(G_1(r)) = \frac{1}{G_1(r)}$ , then we have

$I(G_1(r)) > 1$ , and multiple packets can come in this period and more than one packet can survive in idle time. If we consider one packet has been transmitted from area 1, then the packets from area 2 that has been generated from 1 unit time before starting busy period until end of busy period, can interfere with mentioned packet. Therefore we divide the cycle (Busy period + Idle period) into two part:  $1 + a + B$  and  $I - 1 - a$  where  $a$  is propagation delay. If we consider  $G_1(r) < 1/(1+a)$ , then  $I > 1 + a$ , and  $I - 1 - a$  is positive. The number of successful packets in  $1 + B + a$  interval is  $P_{1+B}(r, z_0)$  and the number of successful packets in  $I - 1 - a$  interval is  $P_{I-1}(r, z_0)$ . The throughput in this case would be:

$$S_{G_1 \leq 1}(r, z_0) = \frac{P_{1+B}(r, z_0) + P_{I-1}(r, z_0)}{B(G_1(r)) + I(G_1(r))} \quad (16)$$

In the second case  $G_1(r) > 1/(1+a)$  and the number of successful packets would be  $P_{1+B}(r, z_0)$  and throughput in this case will be:

$$S_{G_1 \geq 1}(r, z_0) = \frac{P_{1+B}(r, z_0)}{B(G_1(r)) + I(G_1(r))} \quad (17)$$

Then, throughput with capture is given by:

$$S(r, z_0) = \begin{cases} S_{G_1 \leq 1}(r, z_0) & G_1(r) \leq 1/1+a \\ S_{G_1 \geq 1}(r, z_0) & G_1(r) \geq 1/1+a \end{cases} \quad (18)$$

and the overall throughput, considering spatial distribution of terminals, would be:

$$S(z_0) = \int_0^R S(r, z_0) \cdot f(r) \cdot dr \quad (19)$$

where  $f(r)$  is density function of the mobile terminal distance.

#### IV. Results and Discussions

Fig. 2 shows the throughput when the mobile terminal antenna range is 70% and 100% of the base station coverage range with and without capture phenomenon. The throughput loss of about 20% is observed for both cases as the coverage of the terminal drops from 100% to 70% of the coverage of the AP. In both cases capture effects results in about 10% improvement in the throughput of the system.

Fig. 3 shows the probability of success for  $n$  contending packets, with signal-to-interference ratio range from 1 to 4 and equal coverage for the AP and the mobile terminals. As can be seen there is a minimum probability of success when  $n$  gets larger. Fig. 4 compares the throughput of one cell system with equal coverage and natural hidden terminal (the lowest curve), the throughput with capture effect (middle curve), and the throughput of CSMA.

The natural hidden terminal problem decreases the throughput 50% without capture effects. If the antenna range of the mobile terminal is less than the antenna range of base station, then throughput will decrease more, because the area that can be carrier sensed, decreases. The hidden terminal traffic has been modeled as an aloha traffic and has the worst effect on CSMA traffic. The capture effect can increase throughput in both situation the same (10-15%).

#### IV. Conclusion

The effects of natural hidden terminals and the capture on the throughput of a wireless LAN AP using CSMA protocol was analyzed. An analytical frame work was developed for this analysis and it was shown that the natural hidden terminals problem can potentially decrease the throughput of a CSMA network up to 50%. This degradation is very close to the gain of a CSMA over ALOHA due to sensing the channel. It was also shown that in the presence of natural hidden terminal problem, the capture effects would result in 10-15% improvement in the throughput of the AP.

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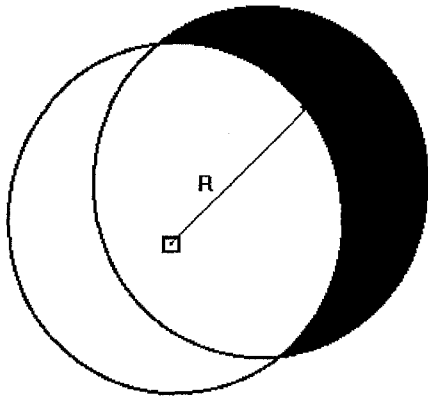


Fig. 1: Natural hidden terminal for wireless APs. The right hand circle represents the coverage area of the AP. The left hand circle represents the coverage area of the terminals in the area designated by the small square. Shaded region represents the area in which the terminals area competing for the same AP but they can not be heard by the terminals in the small square area.

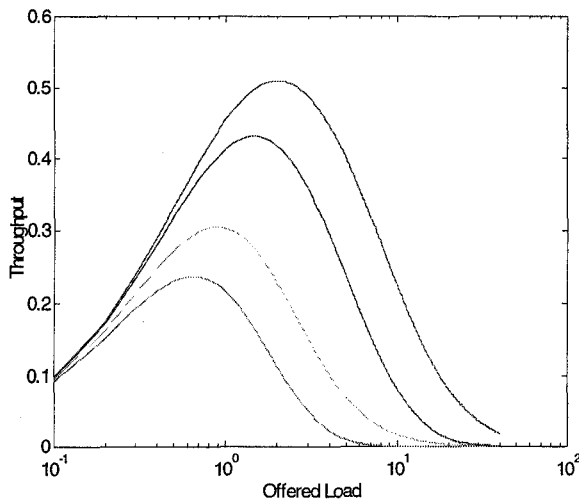


Fig. 2: Effects of the coverage size of the terminal on the overall throughput. The upper two curves represent the throughput with and without capture if the coverage of the AP and the terminals are the same. The lower two curves represent the throughput with and without capture when the range oc coverage of the terminals is 70% of that of the AP.

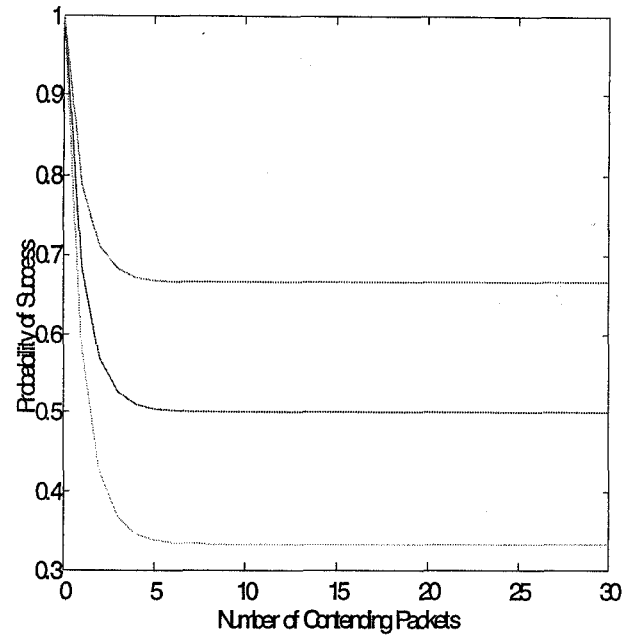


Fig. 3: Probability of success for N contending packets, with uniform terminal distribution for different capture thresholds of  $z_0=4$  (the lower curve)  $z_0=2$  (the middle curve), and  $z_0=1$  (the upper curve).

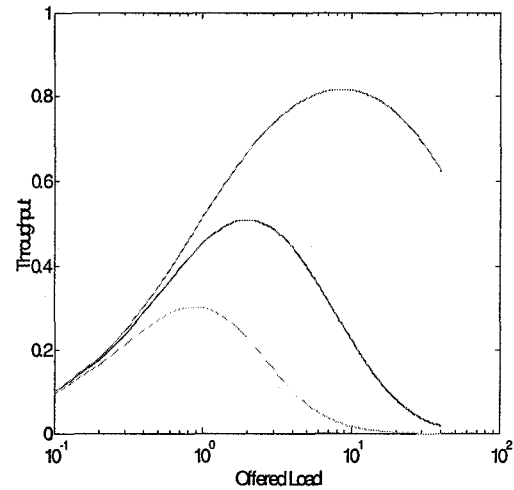


Fig. 4: Throughput of a wireless LAN AP with hidden terminal and capture effect. The upper curve represents the throughput of CSMA as a reference. The lower curve represents the throughput with hidden terminal and no capture. The middle curve corresponds to the case with hidden terminal and capture effects.